University of Sask

Department of Mathematics and Statistics

Math 225 Spring 2003

JUNE 13 2003 | Test #2 | 2 hours
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The examination consists of two parts, Part A and Part B, each worth 20 points. The points however, will be used in a formula to calculate your test grade.

- *Encode* your student number correctly on your opscan sheet.
- *Print* your name and student number on your opscan sheet.
- Answer all questions of Part A ***in pencil*** on your opscan sheet. There is no penalty for a wrong answer in Part A.
- Answer all questions in Part B in the answer book provided.
- One formula sheet is permitted. No calculators.

PART A

Fill in the bubbles on your opscan sheet corresponding to the correct answers. Each problem in this section is worth $1\frac{1}{3}$ points.

Question 1. If $z = st$ and $s = xy$, $t = x^2 + y^2$, then $\dfrac{\partial z}{\partial x}$ is

(A) x (B) t (C) $yt + 2xs$ (D) $2yt + 4xs$ (E) $s + t$
(F) y (G) $yt - xs$ (H) s

Question 2. The tangent plane to the surface $x^2 + z^3y^2 + z^2 = 2$ at the point $(1, 0, 1)$ is

(A) $x - z = 0$ (B) $2x + y - 2z = 0$ (C) $x + 2y - 1 = 0$ (D) $x = 1$ (E) $z = 1$
(F) $x + z - 2 = 0$ (G) $x - y = 1$ (H) $y = 0$

Question 3. If $w = 2s^2 + t^3 + r^2$, $s = xz + y$, $t = 2x^2 + yz$, $r = x^2 + z^2$, then $\dfrac{\partial w}{\partial z}$ is

(A) $4s + 3t^2 + 2r$ (B) $4s$ (C) 0 (D) t (E) $2x + 2z$
(F) $4xs + 3yt^2 + 4zr$ (G) x (H) $2xs^2 + 2zt^3 - yr^2$

Question 4. If $z^3 - xz + 2y^2 = 1$ defines a function $z(x, y)$, then $\dfrac{\partial z}{\partial x}(0, 0, 1)$ is

(A) 1 (B) $\frac{2}{3}$ (C) 2 (D) $\frac{1}{3}$ (E) -1
(F) $\frac{4}{5}$ (G) 0 (H) $\frac{1}{2}$

Question 5. If $f(x, y, z) = x^3 - 4xy^2 + 2e^{xyz}$, then $\nabla f(1, 0, 1)$ is

(A) $(1, 1, 0)$ (B) $(1, 2, 0)$ (C) $(2, 1, 0)$ (D) $(2, 2, 0)$ (E) $(3, 1, 0)$
(F) $(3, 2, 0)$ (G) $(4, 1, 0)$ (H) $(4, 2, 0)$

Question 6. A flea on the (x, y)-plane is standing at the point $(1, 2)$. The temperature in the plane is $100x(y + 1)$. The flea feels too cold and travels, in the most efficient way it can, with aim to warm itself up. The unit vector in the direction along which the fly embarks is

(A) $(1,0)$ (B) $(300,100)$ (C) $(1,1)$ (D) $(1,4)$ (E) $(0,1)$
(F) $(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$ (G) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ (H) $(100,100)$

Question 7. If $\vec{u} = (3, -2)$ and $f(x, y) = x^3 + xy$, then the directional derivative $D_{\vec{u}}f(1, -1)$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2
(F) 3 (G) 4 (H) 5

Question 8. The function $f(x, y, z) = xy + 4z^2$ has a critical point at $(0, 0, 1)$ when it is constrained to the level set $g(x, y, z) = x^2 + 3y^2 + z^2 = 1$. The Lagrange multiplier λ at this critical point is

(A) -1 (B) 0 (C) 1 (D) 2 (E) 3
(F) 4 (G) 5 (H) 6

Questions 9, 10, and 11 refer to functions f through n, which satisfy the following conditions:

$D = AC - B^2$

A	B	C	D
$f_{xx}(0,0) = 8$	$f_{xy}(0,0) = 2$	$f_{yy}(0,0) = -1$	$-$
$h_{xx}(0,0) = 2$	$h_{xy}(0,0) = 0$	$h_{yy}(0,0) = 2$	$+$
$g_{xx}(0,0) = 1$	$g_{xy}(0,0) = -2$	$g_{yy}(0,0) = 4$	0
$k_{xx}(0,0) = -2$	$k_{xy}(0,0) = -1$	$k_{yy}(0,0) = -1$	$+$
$l_{xx}(0,0) = 3$	$l_{xy}(0,0) = 4$	$l_{yy}(0,0) = 0$	$-$
$m_{xx}(0,0) = -1$	$m_{xy}(0,0) = -2$	$m_{yy}(0,0) = 3$	$-$
$n_{xx}(0,0) = -1$	$n_{xy}(0,0) = 1$	$n_{yy}(0,0) = -2$	

Question 9. By the second derivative test, the functions in the above table that have a local maximum at $(0, 0)$ are $D > 0,\ A < 0$

(A) f, g, n (B) h, l (C) m, l (D) k, n (E) g, m, n
(F) f, l, m (G) h (H) g

Question 10. By the second derivative test, the functions in the above table that have a local minimum at $(0, 0)$ are

(A) f, g, n (B) h, l (C) m, l (D) k, n (E) g, m, n
(F) f, l, m (G) h (H) g

Question 11. By the second derivative test, the functions in the above table that have a saddle point at $(0, 0)$ are

(A) f, g, n (B) h, l (C) m, l (D) k, n (E) g, m, n
(F) f, l, m (G) h (H) g

Question 12. The function $z = x^2 + xy + 8y - x$ has a single critical point. That critical point is at

(A) $(-8,17)$ (B) $(-6,21)$ (C) $(0,0$ (D) $(4,11)$ (E) $(0,5)$
(F) $(-5,12)$ (G) $(0,1)$ (H) $(2,13)$

Question 13. The value of $\int_{[0,1]\times[0,1]} x^2 + y^2\, dA$ is

(A) 0 (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) 1 (E) $\frac{4}{3}$
(F) $\frac{2}{5}$ (G) $\frac{1}{3}$ (H) 2

Question 14. The value of $\int_0^1 \int_0^{x^2} xy\, dy\, dx$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{5}$
(F) $\frac{1}{12}$ (G) $\frac{1}{15}$ (H) $\frac{1}{20}$

Question 15. The value of $\int_0^2 \int_{y^2}^{y} 1 + x^2 y\, dx\, dy$ is

(A) $\frac{8}{3}$ (B) $-\frac{46}{5}$ -9 (C) $\frac{7}{3}$ (D) $\frac{6}{7}$ (E) $\frac{4}{3}$
(F) $-\frac{91}{15}$ (G) $\frac{19}{3}$ (H) $-\frac{27}{5}$

PART B

Show all your work in the booklets provided.

Question 16. Evaluate the following limits, with proof, or show that they do not exist: **4**

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos x - \sin y}{x+y} \qquad \lim_{(x,y)\to(0,0)} \frac{x^3 \cos x}{x^2+y^2}$$

Question 17. If $u = f(s,t)$ and $s = \dfrac{x-y}{z}$, $t = \dfrac{1}{y} - \dfrac{1}{x}$, then show that u satisfies **5**

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z(x+y)\frac{\partial u}{\partial z} = 0$$

Question 18. Using the method of Lagrange multipliers, find the critical points of the function $f(x,y) = x^2 - 10x + y^2 - 14y + 70$ when this function is constrained to $x + y = 10$. **5**

Question 19. Find and classify the critical points of the function $f(x,y) = 2xy - \frac{1}{2}(x^4 + y^4) + 3$. **6**